Vector Space and subspace

Consider the vector space $V = \mathbf{R}^3$.

subspace.

- Let U consist of all vectors in R^3 whose entries are equal

$a+b=c$

$k(a)=ka$

$\begin{bmatrix}1\\1\\1\end{bmatrix}$ $\begin{bmatrix}2\\2\\2\end{bmatrix}$ $\begin{bmatrix}-3\\-3\\-3\end{bmatrix}$ $\begin{bmatrix}a\\a\\a\end{bmatrix}$

$$\begin{bmatrix}a\\a\\a\end{bmatrix}+\begin{bmatrix}b\\b\\b\end{bmatrix}=\begin{bmatrix}a+b\\a+b\\a+b\end{bmatrix}$$

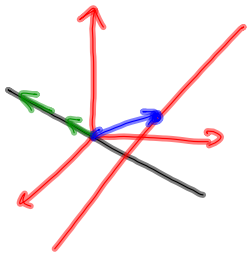
$$k\begin{bmatrix}a\\a\\a\end{bmatrix}=\begin{bmatrix}ka\\ka\\ka\end{bmatrix}$$

x-axis

$\{(x,y,z) \mid x=a,\ y=z=0,\ a\in\mathbb{R}\}$

$(a,0,0)$ ✓

$\begin{bmatrix}a\\0\\a\end{bmatrix}$ ✓



lines that pass through O.

- Let U consist of all vectors in R^3 whose the third entries are equal to zero.

Subspaces of R^3

- zero vector
- lines that pass 0
- planes
  linear
- R^3

Let $V = \mathbf{R}^3$. Show that W is not a subspace of V, where

(a) $W = \{(a, b, c) : a \geq 0\}$, (b) $W = \{(a, b, c) : a^2 + b^2 + c^2 \leq 1\}$.

$w_1 + w_2 = w_3$

$kw_1 = w_3$

ex) $-(a,b,c)$

$= (-a,-b,-c) \notin W$

b/c $-a \not\geq 0$

$\left(\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)+\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)\right) \notin W$

$3(1,0,0)$

$= (3,0,0) \notin W$

R^4

$P(x) = a + bx + cx^2 + dx^3$

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

A_1 A_2

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} + \begin{bmatrix} e \\ f \\ g \\ h \end{bmatrix} = \begin{bmatrix} a+e \\ b+f \\ c+g \\ d+h \end{bmatrix}$$ ✓

$$K\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} Ka \\ Kb \\ Kc \\ Kd \end{bmatrix}$$

Let $V = \mathbf{P}(t)$, the vector space of real polynomials. Determine whether or not W is a subspace of V, where

(a) W consists of all polynomials with integral coefficients.

(b) W consists of all polynomials with degree ≥ 6 and the zero polynomial.

(c) W consists of all polynomials with only even powers of t.

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